Preface

Application and computation of the fundamental trigonometric functions $sin(x)$, $cos(x)$, and $tan(x)$, where x belongs to an input argument in radian, involve in all spheres of our everyday life directly or indirectly; whereas their derivative functions $csc(x)$, $sec(x)$, $tan(x)$, and $cot(x)$ occur less frequently. Consequently, basic trigonometry functions usually refer to $sin(x)$ and $cos(x)$ as well their key derivation $tan(x)$. Explicitly, sine and cosine functions act as parents in the field of trigonometry.

What would we do with sine and cosine functions? Sine and cosine functions have exactly the same shape and frequency, but the cosine function leads the sine function by 90 degrees of phase. Basically, sine and cosine are graphs of wave forms. Although these basic functions have left no field untouched, the ordinary people do not feel their consequences in everyday affairs. Even a long list of nonmathematical academic disciplines rely on these functions, but the members of those disciplines would be unconscious of the fact. In essence, these functions have a great significance in scientific and technical works. Scientists have been applying them for many centuries for miscellaneous aims. These functions are used in various things that we enjoy so much in our everyday life. They are the basic of the theory of periodic functions. They are applied to illustrate the sound and light waves. A sound engineer needs to know at least the basics of the sine and cosine functions for developing good music to calm us from our hectic and stress full daily life. Any kind of circle or oscillation can be modelled or described using sine and cosine functions perfectly. A natural phenomenon is not an organized event made by human beings, although it would affect them directly and indirectly. Most natural phenomena are comparatively innocuous so far as humans are involved. A few examples of natural phenomena include sunrise, sunset, daily temperature, rise and fall of sea levels, and decomposition. Any phenomena having periodic behavior or wave characteristics can be approximately modeled by sine and cosine functions. They form the base for the analysis of speech, frequency modulation, model of waves on the surface of an ocean, pendulum, a child's swing, motion of an engine's piston-crankshaft, Ferris wheel, ebb and flow,

blood pressure in the heart, hours of daylight throughout a year, variation of shadow length, visible shape of the moon, seasons, and boundless other natural phenomena. Within mathematics, they have maximum applications in calculus and linear algebra. Any arbitrary, repetitive mathematical function can be represented by a series of sinusoids of various amplitudes by the process of the Fourier series expansion. It is worth mentioning that the Fourier series cannot be calculated without sine and cosine functions. Besides mathematics, sine and cosine functions are used in astronomy, land surveying and geodesy, navigation on the lands and waters, navigation in space and aeronautic, theory of music, theory of number, theory of probability, theory of game, computer science, Earth science, medical science, forensic science, acoustics, optics, electronics, statistics, phonetics, marketing, electrical engineering, mechanical engineering, civil engineering, physics, chemistry, biology, seismology, meteorology, and oceanography. Henceforth, sine and cosine functions became the inevitably knowing stuffs.

The application disciplines of sine and cosine functions are not only large in number but also distributed. Usually, the usages of sine and cosine functions do not play any attention for general people. To the best of my knowledge, there is no such book that describes many such applications with related computations in one place so that a student or an academician can find ample of resources. This book aims to present selected applications (by categorizing real-world and simulation) as well as computations of sine and cosine functions from miscellaneous disciplines. Predominantly, this book provides a practical, realistic understanding of many and of various types of applications as well as computations of sine and cosine functions. As it might be expected, my choice of application disciplines reflects my own predilections; quite a few other disciplines would have been selected. This book also presents the scientific computational efforts and related results of sine and cosine functions. For example, the computation of *sin(x)* and *cos(x)* simultaneously for any real value of x takes on the average 22.5 nanoseconds for the IA-64 CPU with a clock rate of 1.0 GHz using compiler *gcc* but without its optimizing options, whereas standard math library takes 106.5 nanoseconds for the same specification; resulting a speedup factor of greater than 4. Basically, this book explores the breadth of the subject while including enough depth to convey a good comprehension for the application disciplines involved. Although a significant part of this book is related to computer science and computation related things, it is a multidisciplinary book by nature. This book can be used for academic texts in diverse disciplines.

The primary audience for this book can be anyone, particularly who has ever asked the question of "Where are we ever going to apply the basic trigonometric functions?". Such enthusiast might find this book interesting in reading. To study trigonometry (and also computer architecture) would be a confusing, painful, and joyless endeavor for many students. However, this book gives students an overview of

interrelationship between the basic trigonometric functions (e.g., sine and cosine) and a wide range of academic disciplines in terms of applications and computations. Many trigonometric problems and solutions, which we actually encounter everyday life, have been included in this book. During writing this book, it has been maintained the accessibility for both technical and nontechnical students over a wide range of disciplines as well as educational levels. Besides, it was not the target to make this book mathematical rigor, yet getting students to feel more connected to the basic trigonometric functions a bit more. To a great extent, students who have little mathematical background and a fear of trigonometry courses, may be benefitted from this book. They might be acquainted with the importance of trigonometry in our daily life from the theoretical and simulation point of views. Students who have a strong mathematical education and a high level of motivation, would be benefitted by getting an opportunity to sharpen their reasoning and thinking skills. The student friendly examples (e.g., How does CPU produce the result of $2+3=5$ sequentially?) as well as a clear and seamless writing style can help computer science and engineering students who are reluctant to study computer architecture. For academicians, an attempt has been made to cover miscellaneous implementation and computation of sine and cosine functions in software and hardware with possible examples. Very particularly, it has been included the detailed of possible software optimizations of sine and cosine functions for improving performance. The presented software optimizations are directly tied to the micro-architecture of the specific processors. It has been shown that a careful manual optimization tied closely to the specific processor architecture can provide extensively higher performance than the use of standard math library and hardware division of these functions. While you would very likely find applications and computational methods of sine and cosine functions in this book those you were not aware of; if nothing else, conceivably this book would help with nudging your memory in order to attract attention of trigonometry and computer architecture.

I performed my BSc thesis (in 2001) entitled “Computer simulation of secondary charged particle in a three dimensional radiator”. At first, I got introduced the real use of with the sine and cosine functions in that nuclear physics topic simulation. Yet, I got much insight of the usage of basic functions $1/x$, $1/\sqrt{x}$, $sin(x)$ and $cos(x)$, when I had been working (in 2002-2004) on a project at Max Plank Institute of colloid and interface in Potsdam to develop some efficient and effective tools compatible in Alpha processors for molecular dynamic simulation under the supervision of Dr. Seidel. Later an extended amount of work had been performed on these functions on the updated Alpha processors and Intel architecture during my MSc thesis (in 2004-2006). Since then more than a decade had been passed, I had been appointed as an Assistant Professor at International University of Sarajevo (IUS) in the capital city Sarajevo, Bosnia and Herzegovina. The academic environment over there had influenced me greatly

to write a book. Then I had been looking for a suitable topic to write a book. At the end, I had come to a conclusion that it might have been better to write a book concerning the applications of sine and cosine functions; as I had been gained sufficient practical and theoretical experience on this subject. Besides, computational complexities of sine and cosine functions, a long list of propositions made me more inclined to choose this subject. For examples, (i) Our walking pattern is just a cosine function; (ii) The sites of key famous historical ruins make approximately a sine wave; (iii) The Sun motion controls the shadow length of any object; (iv) Sine wave helps to make prediction; (v) Without adjusting our course of destination using cosine function - it is impossible to reach our destination when we sail in one direction and the current flows in another direction; (vi) The existence of bats depends on sine waves; (vii) Average daily temperature is roughly a sine function; (viii) Sine wave helps to investigate malfeasance; (ix) The Hilbert sine wave discerns between cyclical and trending price activities; and finally, (x) Every kind of regular (i.e., occurring at intervals) behavior can be modeled to some degree of accuracy by straightforwardly amalgamating sine functions. Yet, since then using my leisure time writing routine for a couple of years in a discontinuous manner this book finally came into being.

In a nut shell, primarily this book is devoted to the applications and computations of the parents trigonometric functions namely $sin(x)$ and $cos(x)$. It is organized as follows. Chapter 1 discusses the genesis of basic trigonometric functions, identities, and series. Chapter 2 illustrates miscellaneous applications of sine and cosine functions in our daily life. Chapter 3 investigates several examples of the applications of sine and cosine functions in computer simulation. Chapter 4 scrutinizes various algorithms for computing sine and cosine functions. Chapter 5 provides essential information about computer architecture with examples of basic operations, which would boost both students and academicians in a different way to get deep insight of computer operation. Chapter 6 stands for optimization of sine and cosine functions from hardware point of view. Chapter 7 inspects closely and thoroughly the optimization of sine and cosine functions from software point of view. Appendix A briefs the cardinal information of the processors, which had been used during sine and cosine optimizations. Appendix B presents the answers of selected questions from all chapters.

With best regards,
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